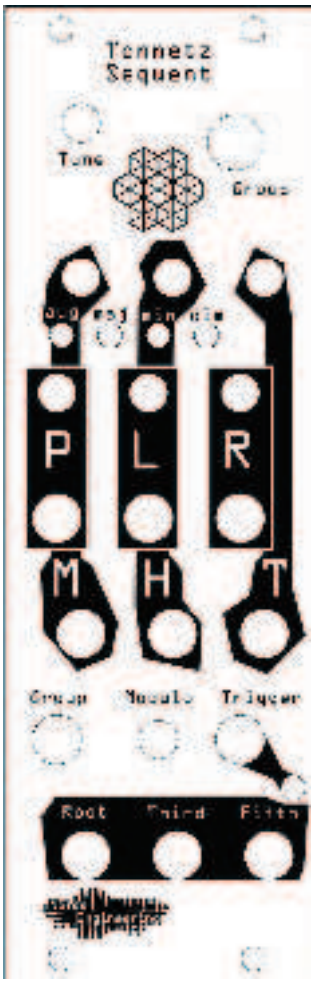
Noise Engineering

Tonnetz Sequent

Eularian Tonnetz Gate-Driven Triad Generator



Overview

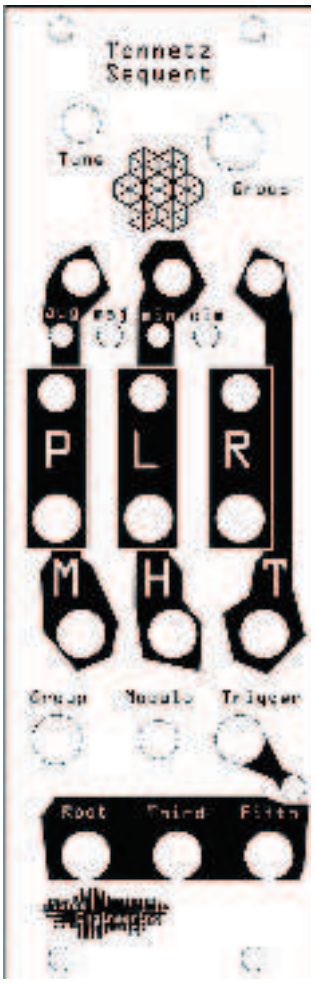
Type	Triad Generator
Size	8HP Eurorack
Depth	.8 Inches
Power	2x5 Eurorack
+12 mA	50
-12 mA	5

Tonnetz Sequent is a triad generator that maps gate inputs to the triadic transforms of the Eularian Tonnetz allowing one to move through triadic space with rhythm. It outputs three - 1v/8va control voltages to control the pitch of oscillators. It has three gate inputs for the standard Tonnetz transforms: Parallel, Leading, and Relative. There are also inputs to reset the triad to a reference triad, perform semitone transpose and change which triad types form the transform domain (major/minor, augmented/major, diminished/minor).

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Patch Tutorial

Pick three oscillators. Connect each pitch out (Root, Third, Fifth) of Tonnetz Sequent to one pitch input of each oscillator. Depress the TUNE button and use the pitch controls on the oscillators to adjust their pitches to unison. Turn the Group knob fully CCW.

Use the PLR buttons to explore the Eulerian Tonnetz. Each time you hit a button it will change the triad to a musically related triad.

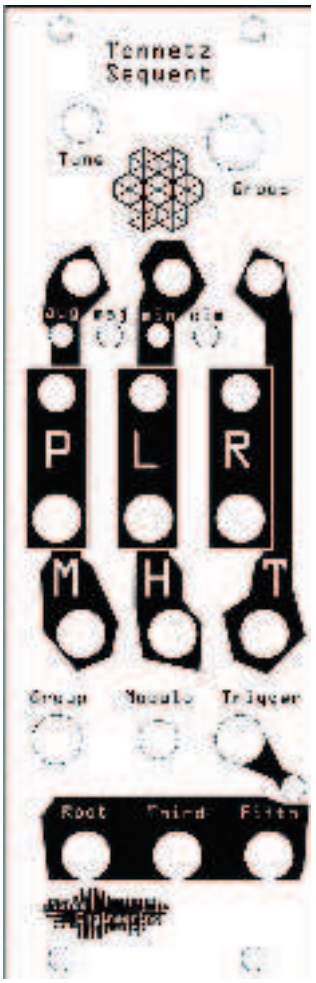
Gates can be input to control the transforms. Connecting three rhythm outputs, perhaps from a Zularic Repetitor, to the PLR jacks will generate a triadic sequence.

To reset the sequence to the reference triad, hit the H button or trigger the H input.

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Interface

Root (pitch output 1v/8va)
Outputs the pitch of the root note of the triad.

Third (pitch output 1v/8va)
Outputs the pitch of the middle note of the triad, typically a third above the root.

Fifth (pitch output 1v/8va)
Outputs the pitch of the top note of the triad, typically a fifth above the root.

P (button, gate input)
Parallel Transform. When in the major mode. P moves the third down a semitone; when in minor P moves the third up a semitone.

L (button, gate input)
Tone Exchange Transform. When in the major mode, L moves the root down a semitone; when in minor, L moves the fifth up a semitone.

R (button, gate input)
Relative Transform. When in the major mode, R moves the fifth up a tone; When in minor, R moves the root down a tone.

M (button, gate input)
Mode transform; changes which pair of triad sets that the transforms act upon. It switches between minor/major, major/augmented, augmented/major, major/minor, minor/diminished, diminished/minor and then back to the start.

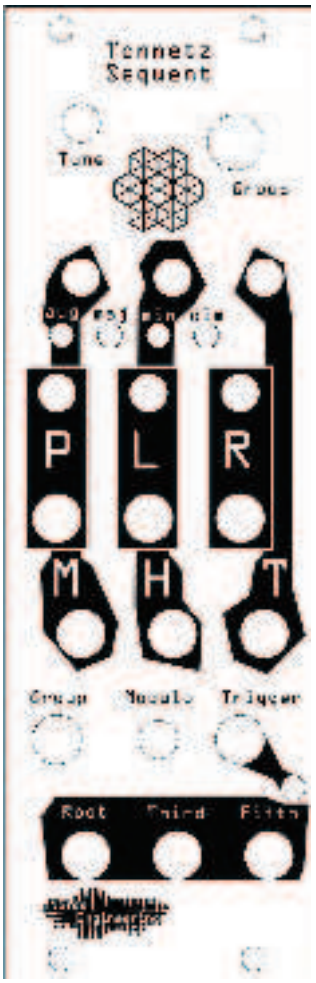
H (button, gate input)
Home transform; changes the current triad back to the reference triad for the current Mode pair.

T (button, gate input)
Transpose transform. Transforms all notes of the triad one semitone up.

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Interface

Group (knob, cv jack input 0-8v)

The group control changes the structure of the transforms by adding a dihedral rotation to P, L and R. The knob and cv input sum together. When fully CCW PLR are canonical.

Modulo (switch)

When enabled triads are constrained within one octave. Pitches that would fall outside of the octave are mapped to their equivalent tone class within the base octave.

Gate(jack gate output)

The Gate output will generate a 20ms pulse every time any transform input occurs (switch or gate). It is essentially a trigger combiner for all gate inputs.

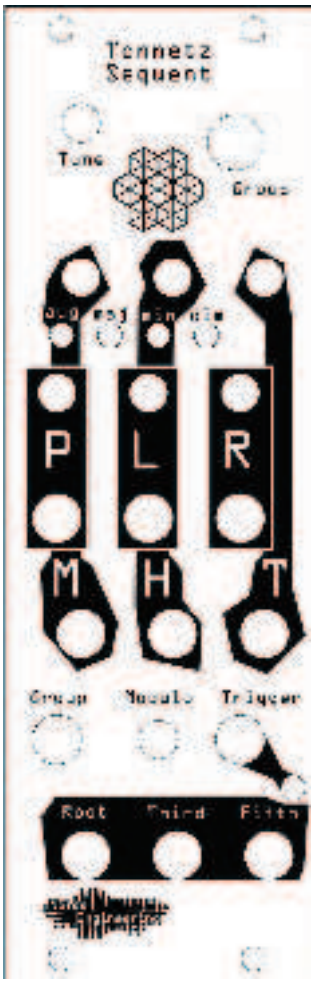
Tune (button)

Depressing the tune button will cause all pitch outputs to be the same as long as it is depressed. This is used to tune connected oscillators .

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Dihedral Transforms

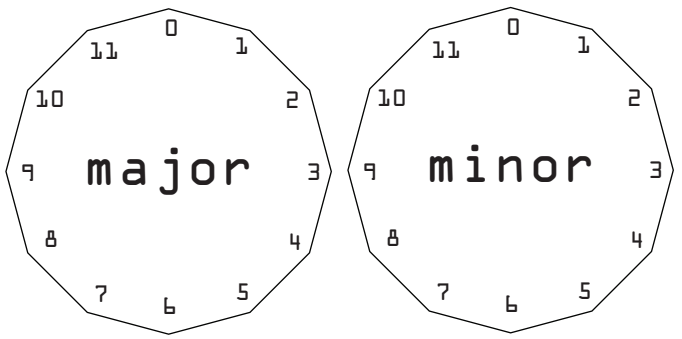
This section will detail the model used for computing triadic transforms used in Tonnetz Sequent. Most users will not need to understand any of the following to use the module but it is provided for those who want to know more details of the implementation. Some abstract algebra knowledge is assumed.

A note on notes: Numeric notation is used for note classes. To map this to standard musical notation let the number 0 be the note class A, the number 1 be the note class A# and so on.

The dihedral group is constructed from two integer modulo groups X and M. M is the group of two elements. X is any additive integer modulo group. The operation of the group is defined by simply composing the elements separately: $(X_0,M_0) * (X_1,M_1) = (X_0*X_1, M_0*M_1)$ for all X_0,X_1 in X and all M_1,M_0 in M. A common interpretation of the dihedral group is rotating and flipping a coin with the orientation of the coin being modeled by X and which side is up being represented by M.

For our discussion M will represent if the triad is major or minor and X is the group of order 12 representing the triads by their root note. For example the element (0,0) is the major triad (0,4,7); the element (1,1) is the minor triad (1,4,8).

Using the coin representation our coin might look like this:

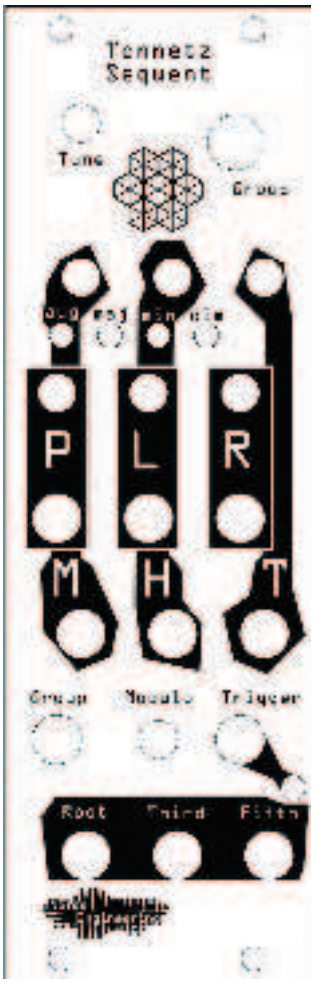


Dihedral Coin

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Dihedral Transforms

Though it takes a bit of work to verify the three parsimonious transforms P, L and R can be represented with this operation by a single number. P is 0, L is 4 and R is 9. This is quite easy to see with P as P takes a triad and maps it to its relative major or minor. L and R are left as an exercise to the reader.

When you hit P, L or R on Tonnetz Sequent internally it does exactly what was previously described. If the current triad is major than the next triad will be minor and rotated according to the number defining the particular transform. Internally subtraction is used for clockwise rotation, addition for counter clock wise. Major triads are denoted 0-11 and minor 12-23 so transforming between major and minor is an offset by 12.

The Group knob modifies this transform by adding its position as an offset; for example if the Group knob is at position 2, then the L transform is now 4+2=6.

This same structure is extended to augmented and diminished triads. At any point in time, Tonnetz Sequent's dihedral coin has either Major/Minor, Major/Augmented or Minor/Diminished on each side.

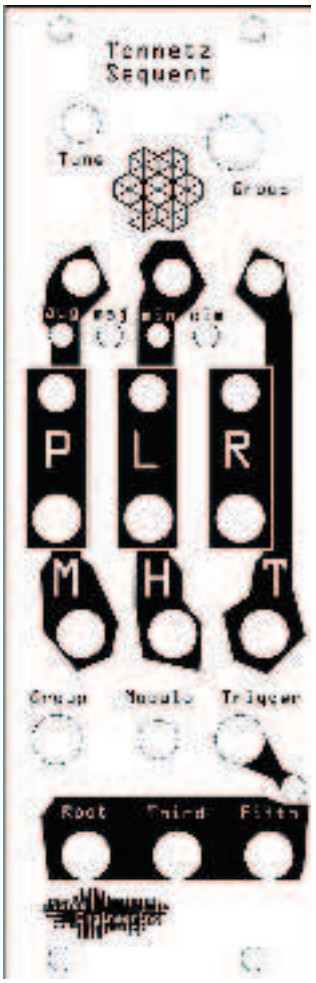
This structure has many different representations; with perhaps the most famous being the Eulerian Tonnetz. In 1739, humanity's greatest mathematician, Leonhard Euler, arraigned the major and minor triads as a triangular grid on the surface of a torus in such a way that motion across the edges of the triangles produces the parsimonious transforms P, L and R. This beautiful structure is, of course, where the name of this module comes from.

Tonnetz Sequent generates triads using gates to determine the direction of each step about the surface of this torus.

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Design Notes

This module came directly out of my interest in Neo-Riemanian music theory. The first design was based around the Eularian Tonnetz (PLR) but as I read more, my design became more abstract, focusing on universal triadic transforms. At some point, I realized that this would make a much less intuitive module and went back to PLR as a basis and added a very simple way of modifying the PLR to produce other transform sets. Similar transform sets to PLR were designed for the diminished and augmented triads.

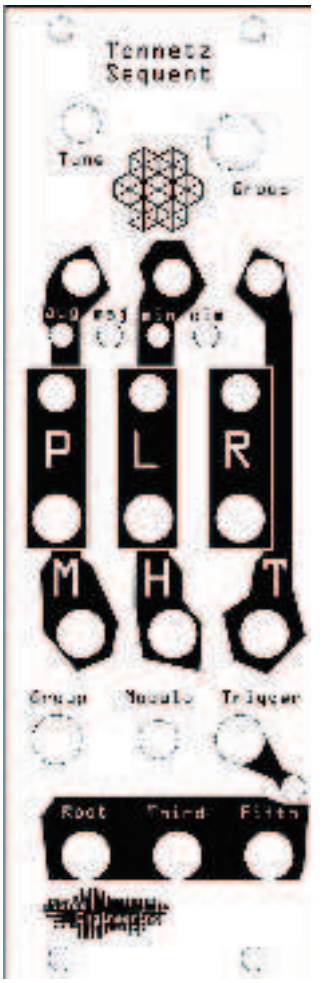
The goal of Tonnetz Sequent is to encapsulate triadic transforms into a module that requires no understanding of the math but just allows the user to intuitively explore the Tonnetz.

On the following page are some references that influenced this module. They are in no means required reading to be able to use this module; they are provided for the curious explorers of music theory.

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Lewin, David. Generalized musical intervals and transformations. Oxford University Press, 2007.

http://en.wikipedia.org/wiki/Tonnetz

http://en.wikipedia.org/wiki/Dihedral_group

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